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At present, the study of solid-propellant ignition is of particular interest owing to the adoption of hybrid motors [1-3]. The status of experimental and theoretical research in this field can be evaluated on the basis of the rather extensive survey of American papers in [2]. It is noteworthy that a common deficiency in available references is the absence of exact ignition criteria; in most cases the propellant is assumed to have ignited when its surface temperature reaches a prescribed level (gasification temperature), or when the rate at which the temperature increases with time at the propellant surface is sufficiently high. Exact criteria for this rate, however, are not given. In this article, we present ignition criteria for solid propellants and these are based on a diffusion-burning model. It is shown that for a diffusion flame to exist above the propellant surface, two conditions must be satisfied simultaneously: 1) the propellant surface temperature must equal the gasification temperature for that propellant and 2) the temperature gradient at the surface must be smaller than some value which depends on the kinetics of the chemical reaction in the diffusion flame and on the rate of oxidizer input to the propellant surface during burning.

Two ignition techniques are examined as examples: ignition by hot gases or radiant heat flow and ignition by means of an active film which reacts with a cold oxidizer; the film is applied to the propellant surface prior to ignition.

1. Conditions necessary for ignition. We start by determining the conditions necessary for the ignition of a solid propellant. We assume that the propellant has ignited when the products of gasification and an oxidizer flowing over the propellant surface forms a diffusion flame above the propellant surface [4]. We assume that the flame is close to the propellant surface, deep inside the boundary layer, so that the motion of the gas near the diffusion flame can be assumed to be one-dimensional. Figure 1 shows a schematic representation of a diffusion flame situated at a distance l_0 from the solid-propellant surface, as well as the distributions of the temperature T , the fuel concentration b , and the oxidizer concentration a at the propellant surface.

In accordance with the assumptions of the Burke-Schumann-Zel'dovich model [5, 6] we assume that the propellant and oxidizer concentrations are zero in the diffusion flame and that their inflows to the flame are in stoichiometric proportion. These conditions are most general and derive solely from the assumption that the kinetic resistance is small as compared with the diffusion resistance.

Both steady-state and transient phases of burning in the diffusion flame above the propellant surface will be examined. Transient is understood to mean the burning phase where the depth of the heated layer in the propellant does not correspond to its burning rate. It will be shown below that, in this case, the temperature in the diffusion flame differs from the steady-state temperature.

The condition of instantaneous adaptation of the processes in the gas phase to the surface state of the

thermal layer in the propellant (a condition widely used in the theory of burning of powders and compacted explosives [7]; a necessary and sufficient condition for its applicability is a sufficiently small ratio of the gas density to the density of the solid material) will be used in the analysis of the heat and mass transfer processes between the surface of the burning propellant and the diffusion flame. In this case, steady-state relations for burning in the gas phase can also be used for nonsteady burning conditions.

We write the condition for fuel-flow conservation between the surfaces $x = -0$ and $x = l_0 + 0$, and take into account the relation between the fuel and oxidizer flows in the diffusion flame, obtaining

$$\rho_1 u_1 = \nu j_1. \tag{1.1}$$

Here, ρ_1 is the propellant density, u_1 is the burning rate, j is the oxidizer flow to the diffusion flame, and ν is the stoichiometric ratio. The mass flow of the oxidizer to the flame is defined by the hydrodynamic conditions of the oxidizer flow past the burning propellant. Namely

$$j = \beta a_0, \tag{1.2}$$

where a_0 is the oxidizer concentration at a point removed from the propellant, while the mass transfer coefficient β can be calculated from the dimensionless relation between the Nusselt numbers, the Reynolds numbers, the Prandtl numbers, and the ratio of the gas discharge velocity of the propellant to the oxidizer longitudinal flow rate.

Relation (1.1) defines the burning rate of the propellant, which is independent of the state of the thermal layer (i.e., it is the same for steady-state and nonsteady burning).

Let us now write the condition for thermal energy flux conservation

$$\begin{aligned} \nu j (c_1 T_s + Q) - \lambda_1 f &= j_q = \alpha (T_r - T_0), \\ (j &\equiv \partial T / \partial x |_{x=0}). \end{aligned} \tag{1.3}$$

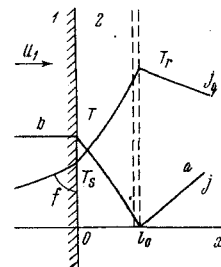


Fig. 1

where c_1 and λ_1 are the specific heat and thermal conductivity of the solid propellant, T_s is its surface temperature, (gasification temperature), f is the temperature gradient at the surface (Fig. 1), Q is the total thermal effect per unit propellant mass, equal to the difference between the thermal effect of the reaction of the products of gasification with the oxidizer in the diffusion flame and the gasification heat of the propellant, and j_q is the heat released by the diffusion flame to the gas flow, which is defined by the heat-transfer coefficient α , calculated from the hydrodynamic conditions of the gas flow past the propellant and also by the difference between the diffusion-flame temperature T_r and the external flow temperature T_e .

For steady-state burning

$$\lambda_1 f^\circ = \nu j c_1 (T_s - T_0) \quad (1.4)$$

(the superscript \circ refers to quantities in the case of steady-state burning), while, from (1.3), the steady-state flame temperature is defined as

$$\alpha(T_r^\circ - T_e) = \nu j (c_1 T_0 + Q) = \nu \beta a_0 (c_1 T_0 + Q) \quad (1.5)$$

(as distinct from the diffusion flame examined by Zel'dovich [6], the temperature of steady-state burning is here, generally speaking, not equal to the burning temperature of a stoichiometric fuel-oxidizer mixture, but rather is defined by the relationship between the intensities of heat and mass transfer at the propellant surface. An analogous situation arises, for example, during catalytic oxidation of hydrogen on a platinum filament, where, owing to the difference between the coefficients of diffusion and thermal diffusivity, the filament temperature differs appreciably from the combustion temperature [5]). Using the temperature T_r° , we introduce the dimensionless variables

$$\theta = \frac{T - T_0}{T_r^\circ - T_0}, \quad \xi = \frac{\nu j c_1}{\lambda_1} x. \quad (1.6)$$

Relation (1.3), which also holds for nonsteady burning, takes the form

$$\varphi^\circ - \varphi = m(\theta_r - 1) \quad \left(m = \frac{\alpha}{\nu j c_1}, \quad \varphi^\circ = \theta_s = \frac{T_s - T_0}{T_r^\circ - T_0} \right) \quad (1.7)$$

where φ is the dimensionless temperature gradient at the surface, φ° is the gradient for steady burning, and θ_r is the nonsteady temperature of the diffusion flame.

Relation (1.7) defines the diffusion flame temperature for a given state of the heated layer in the propellant. For values of gradient φ higher than the steady-state value, the flame temperature is below the steady-state temperature, and vice versa.

In the zone between the propellant surface and the diffusion flame, there occurs a Michelson temperature profile

$$\theta_2 = \frac{1}{1 - e^{-\gamma \xi_0}} [(\theta_s - \theta_r e^{-\gamma \xi_0}) + (\theta_r - \theta_s) e^{\gamma(\xi - \xi_0)}], \quad (1.8)$$

$$\gamma = \frac{c_2}{c_1}, \quad \xi_0 = \frac{\nu j c_1 l_0}{\lambda_1},$$

where γ is the ratio of the specific heats for the gas and solid propellant.

The distance between the diffusion flame and the propellant surface is determined from the condition of contact between the heat flows at this surface

$$\rho u (c_1 T_s - q) - \lambda_1 \frac{\partial T_1}{\partial x} = \rho u c_2 T_s - \lambda_2 \frac{\partial T_2}{\partial x} \quad (1.9)$$

$$(\rho_1 u_1 = \text{const} = \rho_2 u_2 = \rho u).$$

where q is the gasification heat of the fuel, and subscripts 1 and 2 refer to the propellant and gas, respectively. In dimensionless variables, this condition takes the form

$$(1 - \gamma)(\theta_s + \theta_0) - \Delta - \varphi = \frac{\partial \theta_2}{\partial \xi} \Big|_{\xi=0} \quad (1.10)$$

$$\left(\Delta = \frac{q}{c_1 (T_r^\circ - T_0)}, \quad \theta_0 = \frac{T_0}{T_r^\circ - T_0} \right).$$

Substituting the derivative calculated from (1.8) into (1.9), we get

$$e^{\gamma \xi_0} = 1 + \frac{\gamma [(\theta_s - \varphi) / m + 1 - \theta_s]}{(\theta_s - \theta_0)(1 - \gamma) - \Delta - \varphi}. \quad (1.11)$$

For steady-state burning $\varphi = \varphi^\circ = \theta_s$, we have

$$e^{\gamma \xi_0^\circ} = 1 + \frac{\gamma(1 - \theta_s)}{(\theta_s - \theta_0)(1 - \gamma) - \Delta - \theta_s}. \quad (1.12)$$

Let us now examine the behavior of the diffusion flame as the temperature gradient increases. The flame temperature will drop until the oxidizer input (defined by the hydrodynamic conditions) to the flame finally corresponds, for a given flame temperature, to the critical mass flow to the diffusion flame, and which therefore will be extinguished.

We know from theoretical papers [6, 8, 9], verified experimentally, that the critical mass flow to the diffusion flame is defined by the kinetics of the reaction in the flame

$$j_* = k \sqrt{\rho_2 D} \exp - \frac{E}{2RT_r}. \quad (1.13)$$

Here E is the activation energy of the chemical reaction; D is the diffusion coefficient; k is a dimensionless factor which includes the total order of the reaction μ ($k \sim p^{1/2\mu}$, where p is the pressure). The order of magnitude of j_* is the same as that of the mass velocity of flame propagation in a stoichiometric fuel-oxidizer mixture [6, 8].

We define the critical temperature gradient φ_* in the propellant as the gradient for which a given oxidizer input j to the diffusion flame will equal the critical input, due to the drop in temperature T_r . We get

$$\varphi_* = \theta_s + m \left(1 + \theta_0 - \frac{E}{2R(T_r^\circ - T_0) \ln(k \sqrt{\rho_2 D} / j)} \right). \quad (1.14)$$

Thus, to ignite a solid propellant, i.e., to create conditions under which a diffusion flame could exist above the propellant surface, the propellant must be heated to the gasification temperature and, in addition, a temperature gradient, smaller than the critical gradient, must be created at the propellant surface. (As distinct from the ignition of powders and explosives [7, 10], for which the surface of the material must also be heated to a certain temperature and a sufficiently thick boundary layer must be created in the material; the temperature gradient here is defined not only by the thermophysical and kinetic characteristics of the material, but also by the rate at which the oxidizer is supplied to the propellant during burning.) For large values of j , the gradient calculated from formula (1.14) may be smaller than its steady-state

value. Such a propellant cannot be ignited for any mass supply rate.

Let us determine the ignitability condition for propellants by equating the critical and steady-state gradients; the condition for critical mass flow j^* to the propellant surface is obtained in the form

$$1 + \frac{\nu j^* (c_1 T_0 + Q)}{\alpha T_e} = \frac{E}{2RT_e} \frac{1}{\ln(k\sqrt{\rho_2 D}/j^*)}. \quad (1.15)$$

For the determination of j^* , it is convenient to solve this relation graphically (an example of such a solution is given in Fig. 2; the values of $\lg[1 + \nu j^* (c_1 T_0 + Q)/\alpha T_e]$ (curve 1) and $\lg[E/2RT_e \ln(k\sqrt{\rho_2 D}/j^*)]$ (curve 2) are plotted on the y-axis versus j^* for $\nu[c_1 T_0 + Q]/\alpha T_e = 3.5 \cdot 10^3 \text{ cm}^2 \cdot \text{sec/g}$, $E/2RT_e = 10$, and $\ln(k\sqrt{\rho_2 D}) = -0.58$. The point of intersection corresponds to $j^* = 0.52 \text{ g/cm}^2 \cdot \text{sec}$).

It is noteworthy that the impossibility of igniting the propellant is equivalent to the impossibility of a diffusion burning regime. The possibility of a kinetic burning regime, where the chemical reaction takes place within the volume of the combustion chamber, is not excluded when the gas temperature above the propellant is sufficiently high.

2. Propellant ignition by heat flows (hot gases, radiation). The theory of ignition of powders and explosives [7, 10] can be applied in full extent to the ignition of solid propellants, the only difference being that the critical temperature gradient for solid propellants has a different physical meaning. Moreover, the assumptions made in [7, 10] concerning the nature of propellant gasification are more applicable to the conditions of solid propellant ignition, since for solid propellants the gasification reaction is endothermic and cannot lead to self-heating of the propellant.

As in the case of the ignition of powders and explosives [10, 12], several conditions of heat input to the propellant can be distinguished:

1. Weak heat input conditions. The heat flow to the propellant, at the instant that the gasification temperature I_q at the propellant surface is reached, is less than the flow corresponding to the critical gradient:

$$I_q \leq \nu c_1 (T_r^0 - T_0) \Phi_* \quad (2.1)$$

In this case, ignition occurs prior to the establishment of the gasification temperature. The time required for the conformation of a diffusion flame above the surface of an adequately heated propellant is neglected. This time is determined by the heating time of the gas above the propellant, which is small compared with the heating time of the solid material, owing to the low density of the gas. Here, it is necessary that the heat source above the propellant ensure heating of the gas to a temperature on the order of the diffusion flame temperature.

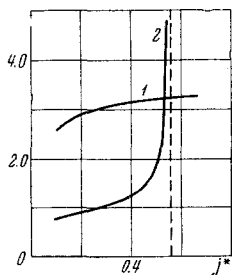


Fig. 2

2. Intense heat input conditions, characterized by

$$\nu j c_1 (T_r^0 - T_0) \Phi_* < I_q \leq \nu j [c_1 (T_s - T_0) + q] \quad (2.2)$$

Under these conditions, the propellant ignites only after gasification of a certain layer of the propellant under the action of the heat flow, following the creation of a critical temperature gradient at the surface.

3. Extremely intense heat input conditions, characterized by

$$\nu j [c_1 (T_s - T_0) + q] < I_q \quad (2.3)$$

Such heat input conditions lead to steady-state gasification where the temperature gradient exceeds the critical gradient, so that diffusion burning cannot occur.

3. Ignition of propellants with active film coatings.

One of the possible means of igniting hybrid propellants is to deposit on the propellant surface a film of a substance capable of actively reacting with a cold gaseous oxidizer in an exothermic reaction. The reaction heat is in part used to heat the propellant, and will eventually lead to its gasification and ignition.

Let us examine the following ignition model. At the initial instant of time ($t = 0$), we have the following temperature and concentration distributions (Fig. 3). For $x < 0$, the solid propellant has a temperature T_0 . On its surface, an active film of thickness δ has been deposited. In the following, it will be assumed that this thickness is much smaller than that of the thermal layer which must be formed in the propellant in order to ignite it. The film thickness will be assumed to be infinitely thin and, therefore, negligible. For $x > 0$, the gas at rest, which contains the oxidizer, is at a temperature T_e ; the oxidizer concentration is a_0 . At the surface of the active film, there is a heterogeneous exothermic reaction which is governed by a strong Arrhenius temperature dependence. Both gaseous and condensed reaction products can form during the reaction.

In view of the abrupt increase in the reaction rate with only a slight increase in the film temperature, the reaction proceeds almost from the start by diffusion. In this case, the rate of heat release is determined by the rate at which the gaseous oxidizer diffuses to the propellant surface.

For a reaction in the diffusion regime, the problem reduces to the solution of the following system of equations:

$$\frac{\partial T_1}{\partial t} = \kappa_1 \frac{\partial^2 T_1}{\partial x^2} \quad \text{for } x < 0 \quad (3.1)$$

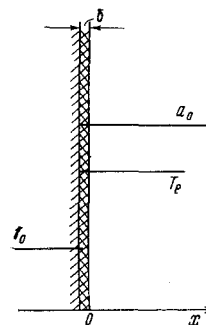


Fig. 3

$$\frac{\partial a}{\partial t} = D \frac{\partial^2 a}{\partial x^2} - u \frac{\partial a}{\partial x}, \quad \frac{\partial T_2}{\partial t} = \kappa_2 \frac{\partial^2 T_2}{\partial x^2} - u \frac{\partial T_2}{\partial x}$$

for $x > 0$ (3.2)

Here, a is the oxidizer concentration, u is the velocity of the gas flow from the propellant; subscripts 1 and 2 refer to the propellant and the gas, respectively; the diffusion coefficient D and the thermal conductivity coefficients $\kappa_{1,2}$ are assumed constant. The boundary conditions are

$$T_1 = T_2, \quad a = 0, \quad \lambda_1 \frac{\partial T_1}{\partial x} - \lambda_2 \frac{\partial T_2}{\partial x} = Q_s D \frac{\partial a}{\partial x} \quad (x=0)$$

$$T_1 = T_0 \quad (x = -\infty), \quad T_2 = T_e, \quad a = a_0 \quad (x = +\infty) \quad (3.3)$$

Here λ is the thermal conductivity and Q_s is the thermal effect of heterogeneous reaction. The outflow velocity u of the gaseous products is related to the diffusion rate by

$$u = \zeta D \left. \frac{\partial a}{\partial x} \right|_{x=+0}, \quad (3.4)$$

where ζ is a coefficient which shows the amount by which the specific volume of the gas changes after the chemical reaction, owing to the change in the mean molecular weight of the gas.

The ignition problem in the case of heterogeneous isothermic reactions at the propellant surface has been examined in American papers [2, 3]. Our solution differs in that it takes into account the influence of the surface temperature gradient on ignition. Neglect of this condition in [2, 3] resulted in an incorrect conclusion that ignition will always take place under kinetic conditions.

Problem (3.1)-(3.3) with boundary conditions (3.4)-(3.5) has a similar solution. Let us introduce the new variables

$$z = \frac{x}{2\sqrt{\kappa_1 t}}; \quad \psi_1 = \frac{T_1 - T_0}{T_0}; \quad \psi_2 = \frac{T_2 - T_0}{T_0}; \quad s = \frac{a}{a_0}. \quad (3.5)$$

Then the equations and the initial and boundary conditions take the form

$$\frac{d^2 \psi_1}{dz^2} + 2z \frac{d\psi_1}{dz} = 0 \quad (3.6)$$

$$\frac{d^2 s}{dz^2} + \left[2L_1 z - \zeta \left(\frac{ds}{dz} \right)_0 \right] \frac{ds}{dz} = 0 \quad \left(L_1 = \frac{\kappa_1}{D} \right); \quad (3.7)$$

$$\frac{d^2 \psi_2}{dz^2} + \frac{1}{L_2} \left[2L_1 z - \zeta \left(\frac{ds}{dz} \right)_0 \right] \frac{d\psi_2}{dz} = 0 \quad \left(L_2 = \frac{\kappa_2}{D} \right); \quad (3.8)$$

$$z = -\infty; \quad \psi_1 = 0;$$

$$z = +\infty; \quad \psi_2 = \psi_e = \frac{T_e - T_0}{T_0}; \quad s = 1;$$

$$z = 0; \quad \psi_1 = \psi_2; \quad s = 0;$$

$$\frac{d\psi_1}{dz} - r \frac{d\psi_2}{dz} = g \frac{ds}{dz} \left(r = \frac{\lambda_2}{\lambda_1}, \quad g = \frac{Q_s D a_0}{T_0 \lambda_1} \right). \quad (3.9)$$

We write the solutions to Eqs. (3.6)-(3.8) under the assumption that $(ds/dz)_0 = \text{const} = n$ (the value of this constant will be determined later).

$$\psi_1 = C_2 + C_1 \text{erf } z,$$

$$s = C_4 + C_3 \text{erf} [l_1 (L_1 z - \zeta n)] \quad (l_1 = 1 / \sqrt{L_1}),$$

$$\psi_2 = C_6 + C_5 \text{erf} [l_1 l_2 (L_1 z - \zeta n)] \quad (l_2 = 1 / \sqrt{L_2}) \text{erf } z \equiv \int_0^z e^{-t^2} dt. \quad (3.10)$$

The integration constants C_i will be determined with the aid of boundary conditions which lead to the relations

$$C_2 = C_1; \quad C_4 + C_5 = 1; \quad C_6 + C_5 = \psi_e;$$

$$C_4 + C_3 \text{erf} (-l_1 \zeta n) = 0; \quad C_6 + C_5 \text{erf} (-l_1 l_2 \zeta n) = 0;$$

$$C_1 - r C_5 \sqrt{L_1 / L_2} e^{-(l_1 l_2 \zeta n)^2} = g C_3 \sqrt{L_1} e^{-(l_1 \zeta n)^2}. \quad (3.11)$$

Finally, we get

$$\psi_1 = \frac{1 + \text{erf } z}{\chi} \left[g \sqrt{L_1} e^{-(l_1 \zeta n)^2} \frac{\text{erfc} (-l_1 l_2 \zeta n)}{\text{erfc} (-l_1 \zeta n)} + \right.$$

$$\left. + \psi_e r \sqrt{L_1 / L_2} e^{-(l_1 l_2 \zeta n)^2} \right]; \quad (3.12)$$

$$s = \frac{1}{\text{erfc}(-l_1 \zeta n)} \{ \text{erf} [l_1 (L_1 z - \zeta n)] - \text{erf} (-l_1 \zeta n) \}; \quad (3.13)$$

$$\psi_2 = \frac{1}{\chi} \left\{ \left[\psi_e - \frac{g \sqrt{L_1} e^{-(l_1 \zeta n)^2}}{\text{erfc} (-l_1 \zeta n)} \right] \text{erf} [l_1 l_2 (L_1 z - \zeta n)] + \right.$$

$$\left. + \frac{g \sqrt{L_1} e^{-(l_1 \zeta n)^2}}{\text{erfc} (-l_1 \zeta n)} + \psi_e [r \sqrt{L_1 / L_2} e^{-(l_1 l_2 \zeta n)^2} - \right.$$

$$\left. - \text{erf} (-l_1 l_2 \zeta n) \right\}; \quad (3.14)$$

$$\chi = r \sqrt{L_1 / L_2} e^{-(l_1 l_2 \zeta n)^2} + \text{erfc} (-l_1 l_2 \zeta n). \quad (3.15)$$

The relation for determining the constant n is

$$n = \left(\frac{ds}{dz} \right)_0 = \frac{2\sqrt{L_1}}{\sqrt{\pi} \text{erfc} (-l_1 \zeta n)} e^{-(l_1 \zeta n)^2}. \quad (3.16)$$

Most convenient is its graphical solution (Fig. 4).

Curve 1 is the right-hand side and curve 2 is the left-hand side of the relation. The parameters have the following values: $L_1 = 1$, $\zeta = 0.3$. The solution for n is $n = 0.92$.

Let us now examine the conditions for the ignition of solid propellants. As stated in section 1, the necessary conditions for ignition are that the surface temperature of the propellant be equal to the gasification temperature and that a sufficiently thick heated layer, characterized by its surface temperature, be developed in the propellant.

As can be seen from the solutions obtained under the assumption of a limiting role for oxidizer diffusion to the propellant surface, the surface temperature of the propellant remains constant during the entire heating process, and is equal to

$$\psi_s = \frac{1}{\chi} \left[g \sqrt{L_1} e^{-(l_1 \zeta n)^2} \frac{\text{erfc} (-l_1 l_2 \zeta n)}{\text{erfc} (-l_1 \zeta n)} + \right.$$

$$\left. + \psi_e r \sqrt{L_1 / L_2} e^{-(l_1 l_2 \zeta n)^2} \right] \quad (3.17)$$

where χ is defined by (3.15).

If this temperature proves to be below the gasification temperature, which we assume to be a known constant, the propellant will not ignite for any thickness of the active film on its surface. However, if the temperature ψ_s is higher than the gasification temperature, ignition will occur beyond the time needed to

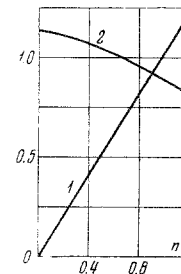


Fig. 4

reach the critical temperature gradient in the propellant. In this case, however, there is the danger that the active film may be blown off by the decomposition products of the solid propellant during the heating process.

Assuming that the film remains on the propellant surface for a sufficiently long time, the minimum time to ignition can be evaluated by calculating the gradient at the propellant surface from formula (3.12) and equating it to the critical gradient

$$t_* = \frac{1}{\pi \lambda_1} \left(\frac{\lambda_1 (T_s - T_0)}{\nu_j c_1 (T_r - T_0) \Psi_*} \right)^2. \quad (3.18)$$

It is of interest to calculate the amount of material (film thickness) required to ignite the propellant. To this end, we determine the amount of oxidizer that has diffused to the propellant surface during the ignition time

$$M = \int_0^{t_*} D \left(\frac{\partial a}{\partial x} \right)_{x=0} dt = \frac{2a_0 \sqrt{D t_*}}{\sqrt{\pi}} \frac{e^{-(t_1 \zeta_n)^2}}{\operatorname{erfc}(-t_1 \zeta_n)}. \quad (3.19)$$

Then the film thickness δ is

$$\delta = \nu M / \rho_0, \quad (3.20)$$

where ρ_0 is the density of the film material.

Let us calculate the efficiency of the film, defining it as the ratio of the quantity of heat expended for heating the solid propellant to the total quantity of heat released in the reaction of the oxidizer with the film material:

$$\eta = \left(\int_0^{t_*} \lambda_1 \left(\frac{\partial T_1}{\partial x} \right)_{x=0} dt \right) \left(\int_0^{t_*} Q_s D \left(\frac{\partial a}{\partial x} \right)_{x=0} dt \right)^{-1}. \quad (3.21)$$

Calculations on the basis of (3.12), (3.13), and (3.16) yield

$$\eta = \frac{L_1 \rho_1 c_1 (T_s - T_0)}{2a_0 Q_s n \sqrt{\pi}}, \quad (3.22)$$

where T_s is defined by formula (3.17).

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